

MA/ MSCMT-10

June - Examination 2019

M.A./M.Sc. (Final) Mathematics Examination**Mathematical Programming****Paper - MA/ MSCMT-10****Time : 3 Hours]****[Max. Marks :- 80**

Note: The question paper is divided into three sections A, B and C. Write answers as per the given instructions. Use of non-programmable scientific calculator is allowed in this paper.

Section - A**8 × 2 = 16**

(Very Short Answer Questions)

Note: Section 'A' contain 08 Very Short Answer Type Questions. Examinees have to attempt all questions. Each question is of 02 marks and maximum word limit may be thirty words.

1) (i) Explain importance of integer programming problem.

(ii) Write the quadratic form

$Q(X) = x_1^2 + 2x_2^2 - 7x_3^2 - 4x_1x_2 + 6x_1x_3 - 5x_2x_3$ in matrix form.

(iii) Write Sylvester's law for definiteness of matrices.

(iv) Define unconstrained optimization problem.

(v) Define extreme point in convex set.

- (vi) State Bellman's principle of optimality.
- (vii) Define convex separable programming problem.
- (viii) Explain duality in quadratic programming problem.

Section - B

4 × 8 = 32

(Short Answer Questions)

Note: Section 'B' contain 08 Short Answer Type Questions. Examinees will have to answer any four (04) questions. Each question is of 08 marks. Examinees have to delimit each answer in maximum 200 words.

- 2) Prove that a hyperplane is a closed set.
- 3) Explain cutting plane algorithm for solving integer programming problem.
- 4) Obtain the necessary and sufficient conditions for the optimum solution of the following Non-linear programming problem.

$$\text{Min. } Z = 4x_1^2 + 2x_2^2 + x_3^2 - 4x_1x_2$$

$$\text{s.t. } x_1 + x_2 + x_3 = 15$$

$$2x_1 - x_2 + 2x_3 = 20$$

$$x_1, x_2, x_3 \geq 0$$

- 5) Solve the following non linear programming problem using the method of Lagrangian multipliers.

$$\text{Min. } Z = x_1^2 + x_2^2 + x_3^2$$

$$\text{s.t. } 4x_1 + x_2^2 + 2x_3 = 14$$

$$x_1, x_2, x_3 \geq 0$$

- 6) Derive the dual of the quadratic programming problem

$$\text{Min. } f(X) = C^T X + \frac{1}{2} X^T G X$$

$$\text{s.t. } AX \geq b$$

Where A is an $m \times n$ real matrix and G is an $n \times n$ real positive semi definite asymmetric matrix.

- 7) Every local maximum of the general convex programming problem is its global maximum.
- 8) Use Bellman's optimality principle to divide a positive quantity 'b' into n parts in such a way that their product is maximum.
- 9) Solve the following L.P.P by using dynamic programming

$$\text{Max. } z = 3x_1 + 5x_2$$

$$\text{S.T. } x_1 \leq 4,$$

$$x_2 \leq 6,$$

$$3x_1 + 2x_2 \leq 18$$

$$x_1, x_2 \geq 0$$

Section - C

$2 \times 16 = 32$

(Long Answer Questions)

Note: Section 'C' contain 04 Long Answer Type Questions. Examinees will have to answer any two (02) questions. Each question is of 16 marks. Examinees have to delimit each answer in maximum 500 words.

- 10) Solve the following linear programming problem using revised simplex method.

$$\text{Max. } Z = 3x_1 + x_2 + 2x_3 + 7x_4$$

$$\text{S.t. } 2x_1 + 3x_2 - x_3 + 4x_4 \leq 40$$

$$-2x_1 + 2x_2 + 5x_3 - x_4 \leq 35$$

$$x_1 + x_2 - 2x_3 + 3x_4 \leq 100$$

$$x_1 \geq 2, x_2 \geq 1, x_3 \geq 3, x_4 \geq 4$$

- 11) Solve the following integer programming problem by branch and bound technique.

$$\text{Max. } Z = x_1 + x_2$$

$$\text{S. t. } 3x_1 + 2x_2 \leq 12$$

$$x_2 \leq 2$$

$$x_1 \geq 0, x_2 \geq 0$$

And are integers.

- 12) Use Kuhn-Tucker conditions to solve the following nonlinear programming problem :

$$\text{Optimize } f(x_1, x_2, x_3) = 2x_1 + 3x_2 - (x_1^2 + x_2^2 + x_3^2)$$

$$\text{Subject to } x_1 + x_2 \leq 1, 2x_1 + 3x_2 \leq 6, x_1 \geq 0, x_2 \geq 0, x_3 \geq 0$$

- 13) Solve the following quadratic programming problem by Wolfe's Method

$$\text{Min. } f(x_1, x_2) = -10x_1 - 25x_2 + 10x_1^2 + x_2^2 + 4x_1x_2$$

$$\text{S.t. } x_1 + 2x_2 \leq 10$$

$$x_1 + x_2 \leq 9$$

$$x_1 \geq 0, x_2 \geq 0$$